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Efficient Transonic Shock-Free Wing Redesign Procedure Using a Fictitious Gas Method

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N.J. Yu*
The Boeing Company, Seattle, Wash.

A new method for transonic shock-free three-dimensional wing design based on the full potential equation is presented. The method locally modifies (i.e., redesigns) the wing surface geometry beneath the supersonic region to produce a shock-free flow. Results for a redesigned nonlifting rectangular wing and for a redesigned lifting wing of ONERA M6 planform are illustrated. Significant drag reduction and improved lift-drag characteristics are achieved from the present redesign procedures. Methods of compensating for wing thickness reduction experienced in the redesign process and of correcting for viscous effects are discussed.

Introduction

OVER the last decade, a major thrust of research in the field of computational aerodynamics has been the analysis and design of transonic configurations. In the analysis area, approaches such as finite difference,[1-4] finite volume,[5] and finite element[6] techniques have been developed. Certain of these have developed to the point of providing accurate and reliable predictions of inviscid surface pressure distributions and lift and, in some cases, useful information concerning drag.

In the area of design methods, extensive research has been carried out for two-dimensional problems—e.g., the hodograph-based works of Nieuwland,[7] Bauer et al.,[8] Boerstoel,[9] and Sobieczky,[10] and the use of optimization techniques (Hicks et al.[11]) for the design of supercritical airfoils without drag creep. Iterative finite difference methods have been explored by Steger and Klineberg,[12] Tranen,[13] Carlson,[14] Shankar et al.,[15] and Volpe,[16] wherein the design problem is treated as an inverse problem, i.e., the airfoil shape that supports a given pressure distribution is found by iteration.

In the three-dimensional design problem, little has been achieved so far. The major difficulties are the high cost of three-dimensional computational explorations and the lack of an elegant method, such as the hodograph method. Hicks and Henne[17] have extended the optimization technique to wing design with substantial success. However, the computational cost of such a design is still relatively high. Moreover, the existence of three-dimensional shock-free configurations has not been proven. However, if proper design techniques are used, approximate shock-free configurations, which are useful from a practical point of view, can be obtained. Sobieczky et al.[18] have extended their fictitious gas method to wing design using a small disturbance formulation. The wing with modified surface slope does provide a nearly shock-free flowfield at the design condition.

In this paper, a computationally efficient and practical design technique based on a solution of the full potential equations is presented. The basic technique modifies a given initial wing geometry to produce a "redesigned" geometry exhibiting shock-free flow characteristics. To demonstrate the validity of the method, the redesigned wings are checked by the analysis code developed by Jameson and Caughey.[5] Results for two redesigned wings are included in the paper.

The present redesign process generally reduces slightly the thickness ratio of the wing. Methods of compensating for this thickness reduction in the redesign procedures and of correcting for boundary-layer displacement thickness effects are illustrated by a two-dimensional example. Some remaining numerical problems in the three-dimensional design procedures, such as numerical stability and accuracy for hyperbolic equations, are also discussed, and methods of resolving such problems are proposed.

Formulation of Governing Equations

The present design study assumes potential flow, i.e., entropy production across shock waves is neglected. The governing equations for mass, energy, and entropy conservation are

$$(\rho\phi_x)_x + (\rho\phi_y)_y + (\rho\phi_z)_z = 0 \tag{1}$$

$$a^2 + \frac{\gamma-1}{2}q^2 = \frac{1}{M_\infty^2} + \frac{\gamma-1}{2} \tag{2}$$

$$\rho = (M_\infty^2 a^2)^{1/(\gamma-1)} \tag{3}$$

respectively, where ϕ is the complete velocity potential. All flow variables are normalized with respect to freestream conditions. For a given wing at given freestream conditions, Eqs. (1-3) can be solved numerically.[3,5] In the present work, it is essential to use a fully conservative numerical solution scheme in order to eliminate surface geometry closure problems caused by unbalanced mass fluxes. This leads to the choice of Jameson-Caughey's finite volume code[5] as a basic program and starting point for our design method.

Fictitious Gas Concept

The principal design concept, as detailed in an earlier work,[18] embodies a modification of governing equations locally in the supersonic region to retain an elliptic behavior within the supersonic bubble—hence, the terminology "fictitious gas." This may be done in a number of ways, but it must be done in such a way to satisfy conservation of mass, both locally and globally. The simplest modification is to set the local density ρ equal to the critical density ρ^* whenever the flow becomes supersonic. That is, the continuity equation in the supersonic region becomes

$$(\rho^*\phi_x)_x + (\rho^*\phi_y)_y + (\rho^*\phi_z)_z = 0 \tag{4a}$$

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*Specialist Engineer, Aerodynamic Research Group. Member AIAA.

or simply

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{4b}$$

With such a modification, the governing equations in the whole flowfield become purely elliptic, and the numerical solution of the modified equations contains no shock-wave-like discontinuities.

Design Method

The present design method begins with the geometry of an initial guess wing that typically produces a shock wave terminating the supersonic bubble. The method then provides a local modification of the wing surface underneath the supersonic bubble so as to produce a shock-free flow. The complete design procedure includes the following three major steps.

A. Fictitious Gas Calculations of Transonic Flows over Initial Input Wing

An analysis problem is carried out over the initial guess wing using the modified governing Eq. (4a) in the supersonic region together with the original Eqs. (1-3) for all subsonic regions. This is done with a modified version of Jameson-Caughey's finite volume code. Because the governing equations are purely elliptic, the relaxation procedure used in the code will always provide a converged solution if relaxation factors are properly chosen. The solution obtained provides a solution to the correct governing equations in all subsonic regions up to the sonic line. The results within the supersonic bubble, where the gas is "fictitious," are discarded. The sonic line data are saved for use in step 2.

B. Recalculation of Supersonic Region Using Real Gas Law and Finding New Wing Shape

The major task of the present design technique are to replace the fictitious gas flow within the supersonic bubble with a real gas flow subject to the boundary data known along the sonic line from step 1 and to calculate a corresponding new wing surface shape beneath the bubble. This is done by utilizing the three irrotational equations,

$$u_y = v_x \tag{5a}$$

$$u_z = w_x \tag{5b}$$

$$v_z = w_y \tag{5c}$$

the continuity equation in quasilinear form,

$$(a^2 - u^2)u_x + (a^2 - v^2)v_y + (a^2 - w^2)w_z$$
$$-2uvu_y - 2vwv_z - 2uwu_z = 0 \tag{6}$$

as well as five Taylor's series expansions of $u, v,$ and w in two directions to solve the nine derivative terms (i.e., u_x, u_y, u_z, v_x, v_y, v_z, and w_x, w_y, w_z) by a marching procedure. Detailed formulations of the difference equations at each computational point are shown in the Appendix.

In the formulation of a marching technique for hyperbolic equations, proper consideration must be taken of the domain of dependence in the calculation procedure. The most accurate and reliable marching technique is the method of characteristics. However, in three-dimensional flows, characteristics methods are too complicated to be practical. We alternatively use an implicit iterative method with proper computational grid alignment, as shown in Fig. 1. The computational region is confined between the sonic surface and a surface connecting the first and the last sonic points at each span station. The x coordinate is equally divided by the same number of grid points at every span station, while in the z direction it is divided into the same number of layers, that is, Δx is a function of y only, but Δz is a function of both x and y.

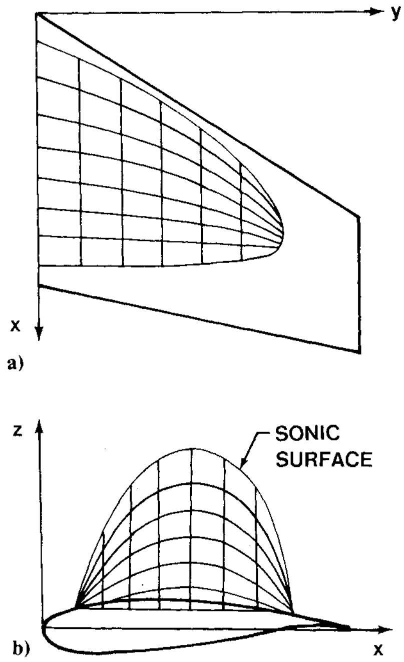


Fig. 1 Computational grid alignment for design calculations.

Such mesh alignment not only provides a suitable domain of dependence for the computation but also gives a better resolution near the rapid expansion and compression regions in the flowfield.

In the actual computations, Eq. (A6) is used to calculate the derivative terms at each new layer. For the first iteration, data on the old level are always used. The flow variables $u, v,$ and w on the new layer are respectively integrated along a marching direction, e.g., z direction, using a trapezoidal rule

$$f_2 = f_1 + \tfrac{1}{2}(f_{z_1} + f_{z_2})\Delta z \tag{7}$$

Notice that the averaged value of f_z at the old and the new layers is used in Eq. (7). The iteration process continues until the flow variables on the new layer attain a prescribed accuracy.

The marching method illustrated here is fully implicit, i.e., the flow variables at each grid point on the new computational layer are fully coupled. Thus, the scheme is unconditionally stable. Numerical experiments indicated that at moderate freestream Mach number and lift coefficient, a well-converged solution which yields a shock-free configuration can usually be obtained from the present redesign procedure. However, special care must be taken in calculating the derivative terms for the marching procedure. During the early stage of the program development, a cubic spline method was used to compute the first derivative terms of $u, v,$ and w. It was found that the numerical values of the first derivative terms oscillated in the regions of rapid expansion and compression, which, in turn, causes instability in the redesign calculations. The use of a Taylor's series expansion in the present study to evaluate the first derivative terms along each computational surface eliminates such instability problems.

It happens quite often that a sonic surface may be quite steep or may even wrap around the leading edge. For example, some modern wings with blunt leading edges and/or large swept angles tend to have a sonic surface starting right at the leading edge and bending upstream slightly. In such cases, calculations based on the aforementioned mesh alignment either yield inaccurate results or break down. To overcome such difficulties, a coordinate transformation or mapping may be used. In the present work a cylindrical coordinate system suggested by Rubbert[19] was implemented. Let

$$x - x_0(y) = r\cos\theta \tag{8a}$$

$$z - z_0(y) = r\sin\theta \tag{8b}$$

where $x_0(y)$ and $z_0(y)$ are the origins of the cylindrical coordinates at each span station. These functions are chosen so that each grid point on the sonic surface becomes a single value function of θ.

The continuity equation and the irrotationality conditions written in the new coordinate system are:

$$[(a^2-u^2)\cos\theta - 2uw\sin\theta]u_r + (a^2-w^2)\sin\theta w_r$$
$$+[(a^2-v^2)r_y - 2vw\sin\theta - 2uv\cos\theta]v_r$$
$$-[2uw\cos\theta + (a^2-u^2)\sin\theta]u_\theta/r + (a^2-w^2)\cos\theta w_\theta/r$$
$$+[(a^2-v^2)\theta_y - 2vw\cos\theta/r + 2uv\sin\theta/r]$$
$$\times v_\theta + (a^2-v^2)v_s = 0 \tag{9}$$

and

$$\cos\theta w_r - \sin\theta w_\theta/r = \sin\theta u_r + \cos\theta u_\theta/r \tag{10a}$$

$$r_y u_r + \theta_y u_\theta + u_s = \cos\theta v_r - \sin\theta v_\theta/r \tag{10b}$$

$$r_y w_r + \theta_y w_\theta + w_s = \sin\theta v_r + \cos\theta v_\theta/r \tag{10c}$$

where

$$r_y = -\cos\theta\frac{dx_0}{dy} - \sin\theta\frac{dz_0}{dy}$$

$$\theta_y = \left[\sin\theta\frac{dx_0}{dy} - \cos\theta\frac{dz_0}{dy}\right]\Big/ r$$

The computational meshes are constructed in a way similar to that of the Cartesian coordinates with r chosen as the marching direction.

After the supersonic region is computed, the new wing shape and pressure distribution associated with it are then calculated. The associated boundary condition is that the new wing surface be a stream surface of the real gas flow within the supersonic bubble. If

$$F = z - g(x,y) = 0$$

denotes the new wing shape, the stream surface condition becomes

$$uz_x - w + vz_y = 0 \tag{11}$$

Introducing a Taylor's series expansion at constant y station (denoted by subscript 1) for the new shape leads to

$$dz_1 = z_x dx_1 \tag{12a}$$

and in any other direction (denoted by subscript 2)

$$dz_2 = z_x dx_2 + z_y dy_2 \tag{12b}$$

Elimination of z_x and z_y from Eqs. (11), (12a), and (12b) gives

$$dz_1 = \left(\frac{w dy_2 - v dz_2}{u dy_2 - v dx_2}\right)dx_1 \tag{13}$$

Equation (13) is used for the integration of the new surface coordinate z at each span station, proceeding downstream from the upstream intersection of the sonic line with the wing surface. At the first integration step, the values of u, v, and w on the original wing surface are used for Eq. (13). For each subsequent integration step, values of u, v, and w are interpolated from the flowfield solution. The integration procedure continues iteratively until the new surface coordinate converges to a given accuracy. The integration formula for cylindrical coordinates can be derived in a similar way and is not repeated here.

C. Check Calculation of Redesigned Wing Using Original Analysis Code

The final step of our design procedure consists in verifying the redesigned wing to insure that it does indeed provide a shock-free or nearly shock-free pressure distribution. The redesigned wing is analyzed by the standard finite volume code, and the calculated pressure distribution is compared with that obtained from the design procedure. The agreement between the two results serves as a check of the validity of the present design method.

Numerical Results

In order to test the present design technique, a nonlifting aspect ratio 4 rectangular wing with an NACA 64A series airfoil was selected. The thickness ratio tapered linearly spanwise from 6% chord at the root to 0.6% chord at the tip (so as to cause the sonic surface to terminate near midspan, a feature selected for early testing of the method). The design Mach number was $M_\infty = 0.91$. Figure 2 illustrates the wing planform and the sonic surface obtained by the fictitious gas calculations. Figure 3 compares the pressure distributions of the original, the redesigned, and the analysis of the redesigned wing at three span stations. The slight discrepancies between the redesigned and the analysis of the redesigned pressure coefficient near the front junction of the wing and the sonic surface are due primarily to numerical discretization errors. The rapid variation on the flow properties at this region produces larger discretization errors in the redesign calculations, which lead to larger errors in the redesigned shape and its corresponding pressure distribution. The shock wave, as well as the wave drag associated with it, has been eliminated by this design process.

In two-dimensional airfoil redesign, Sobieczky et al.[18] proved that a closed and smooth redesigned shape can be obtained if a fully conservative scheme is used in the fictitious gas analysis. In three-dimensional configuration redesign, there is no guarantee of exact closure of the redesigned shape. Instead, it relies on numerical experiments to check the closure. Experience to date indicates that when the redesign calculation converges, a smooth redesigned shape with very minimal closure gaps is usually obtained and gives little problem for practical engineering applications. Here, as an example, a new shape at several x stations is plotted against y and compared with the original shape in Fig. 4. As can be seen clearly, there is no plottable gap where the new shape fairs into the original wing contour at the sonic surface. The design program has also been tested by mesh refinement in the x, y, and z directions separately or simultaneously. It was concluded that finer mesh calculations provide more accurate results.

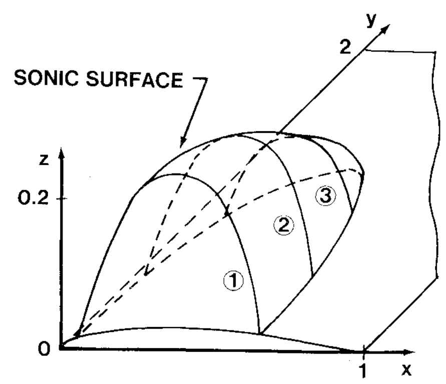
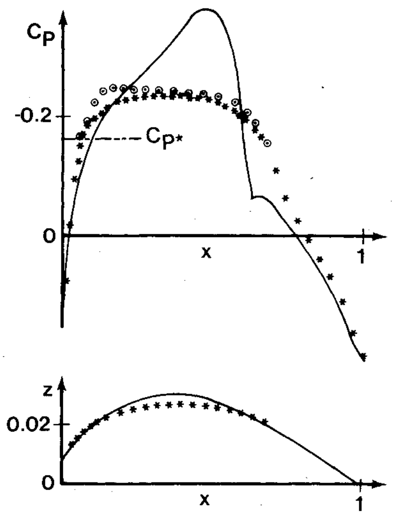


Fig. 2 Sonic surface for nonlifting rectangular wing at $M_\infty = 0.91$, $\alpha = 0$ deg.

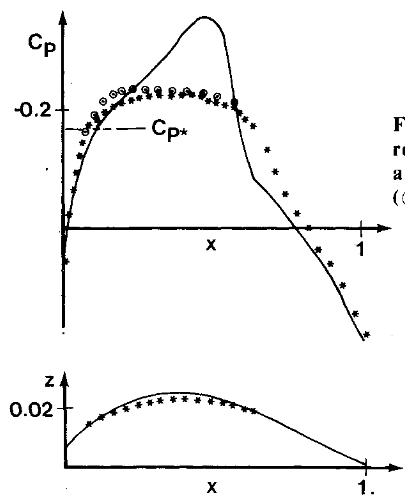
The redesign of a lifting wing is shown in Fig. 5. The original wing has an ONERA M6 planform and an NACA 64A410 airfoil, and was redesigned at $M_\infty = 0.77$ and $\alpha = 3.06$ deg. Here, the sonic surface, Fig. 5, obtained from the fictitious gas calculation extends to the wing tip. The original and the redesigned wing shapes, together with their pressure distributions at several span stations, are shown in Fig. 6.

During this investigation, it was observed that a medium mesh calculation produced a noticeable gap in the redesigned shape in the region near the root and tip. This was attributed to numerical inaccuracies with rapidly changing flow variables in these regions. Subsequent calculations using a refined mesh greatly reduced the problem, indicating that the primary cause was one of numerical resolution.

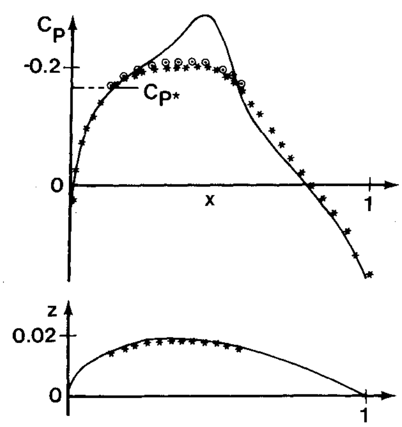
Various methods are under investigation in order to improve the detailed features of the design technique. These include mesh refinement and the use of various gas laws in the fictitious gas calculations.



a. STATION 1, y = 0



b. Station 2, y = 0.4



c. Station 3, y = 0.8

Fig. 3 Original (—), redesigned (***), and analysis of redesigned (⊙ ⊙) results.

Boundary-Layer Correction

In practical airfoil and wing design, one must take viscous displacement thickness effects into consideration. This is especially important for transonic configuration design, since a small error in design shape and/or slope may cause substantial deviation from the expected pressure distribution.

In the present design technique, a method of adding thickness distribution smoothly over the lower surface of the airfoil or wing is proposed.[20] Experience indicates that added thickness over the lower surface has only a small effect on the upper surface pressure distribution. Such added thickness will be used for viscous correction. That is, the displacement thickness calculated by a boundary-layer code will be subtracted from the designed shape. To illustrate the idea, an exponential thickness distribution is added to a two-dimensional airfoil, i.e.,

$$\Delta y = \Delta\tau(1 - e^{-x/b}) \tag{14}$$

where $\Delta\tau$ is the maximum thickness to be added, and b is a constant that controls the thickness distribution.

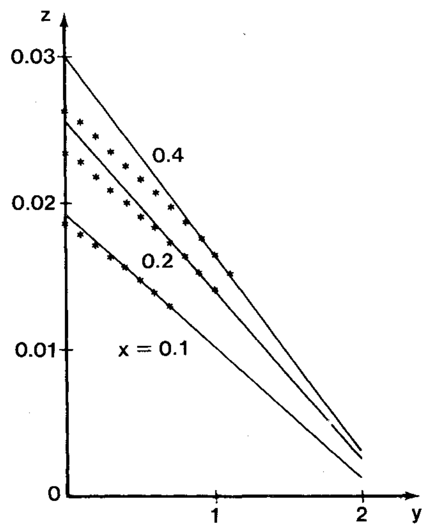


Fig. 4 Original (—) and redesigned (***) spanwise thickness distribution for nonlifting rectangular wing at $M_\infty = 0.91$, $\alpha = 0$ deg.

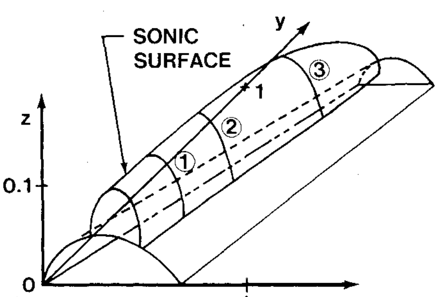


Fig. 5 Sonic surface for lifting wing with ONERA M6 planform at $M_\infty = 0.77$, $\sigma = 3.06$ deg.

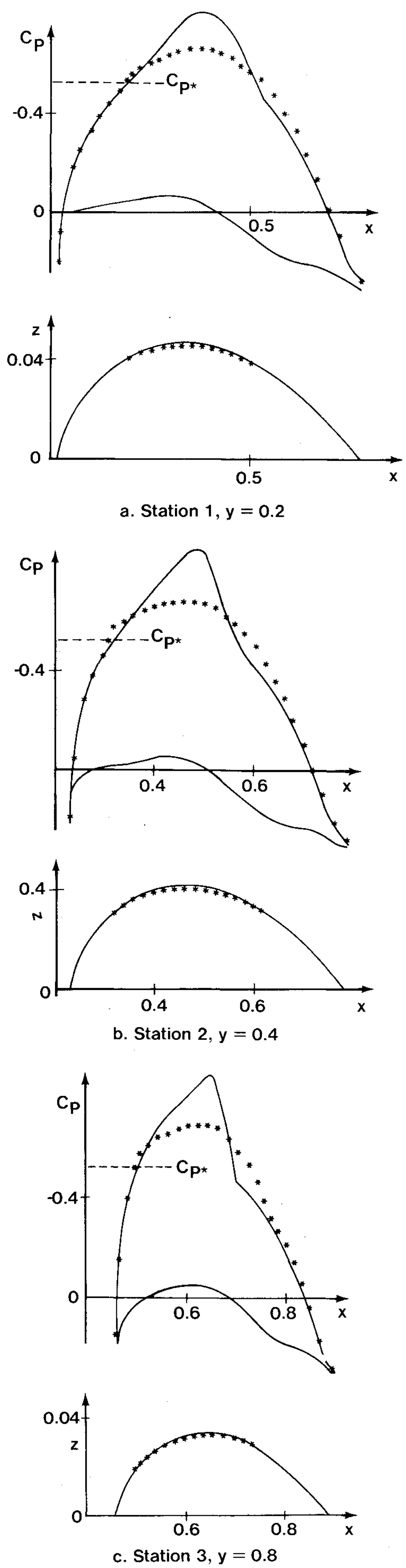


Fig. 6 Original (——) and redesigned (***) results.

The thickened airfoil is redesigned by the present procedure, where the pressure distribution obtained from the design is used for the displacement thickness calculation using a boundary-layer code. Figure 7 illustrates the redesigned NACA 64A410 airfoil with 2% chord thickness added over the lower surface at $M_\infty = 0.72$, $\alpha = 0$ deg, and $Re = 2 \times 10^6$. Here, b is chosen to be a 12.5% chord. The method is also extendable to wing design when it is combined with a three-dimensional boundary-layer code.[21]

The shock-free wing design method presented here tends to reduce the maximum thickness of the original wing. For flows at high freestream Mach numbers or at high lift coefficients, the thickness reduction invoked in the design process may in certain cases be too large to be acceptable. In such cases, one can simply add a certain amount of thickness over the lower surface, and repeat the design procedure so as to obtain a shape that will meet the thickness constraint.

Conclusion

A simple and efficient transonic shock-free wing design method has been developed. Using the method, existing wings have been modified to achieve a shock-free flow condition. The design procedure employed in the study eliminates the shock wave and the wave drag associated with it at the expense of reducing thickness. A method of compensating for thickness reduction as well as of correcting for viscous effects is proposed.

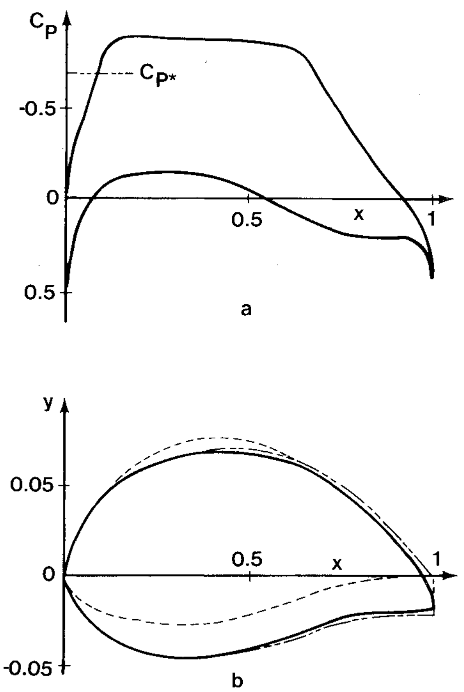


Fig. 7 Original (---), redesigned displacement surface (—·—), and redesigned airfoil (——) at $M_\infty = 0.72$, $\alpha = 0$ deg, and $Re = 2 \times 10^6$.

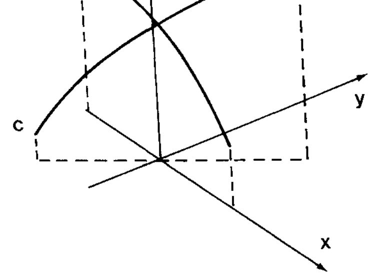


Fig. 8 Grid points on the computational surface.

The results presented in the paper used constant density ρ^* in the supersonic region for the fictitious gas calculations. This is not the only choice one can have for the present design purpose. In fact, a large family of fictitious gases can be used, e.g., a gas with continuous density variation across the sonic surface can also be implemented into the program. Currently, we are evaluating the effects of using different gases on the geometry and the aerodynamic performance of a redesigned airfoil or wing.[22] Further development on this design technique is desirable in order to obtain an optimum design for a given design constraint.

Appendix—Difference Equations for Design Calculation

Consider the mesh alignment in Fig. 8. Let P be the point where the derivative terms are to be found. The Taylor's series expansion of u along aPb, i.e., along a constant y direction is

$$du_1 = u_x dx_1 + u_z dz_1 \tag{A1}$$

and, along cPd is

$$du_2 = u_x dx_2 + u_y dy_2 + u_z dz_2 \tag{A2}$$

where

$$df_1 = f_b - f_a \quad \text{and} \quad df_2 = f_d - f_c$$

Similarly, one can derive

$$dw_1 = u_z dx_1 + w_z dz_1 \tag{A3}$$

$$dw_2 = u_z dx_2 + v_z dy_2 + w_z dz_2 \tag{A4}$$

$$dv_2 = u_y dx_2 + v_y dy_2 + v_z dz_2 \tag{A5}$$

Equations (A1-A5) plus the continuity Eq. (6) can be written in matrix form

$$\begin{bmatrix} dx_1 & dz_1 & 0 & 0 & 0 & 0 \\ dx_2 & dz_2 & dy_2 & 0 & 0 & 0 \\ 0 & dx_1 & 0 & dz_1 & 0 & 0 \\ 0 & dx_2 & 0 & dz_2 & dy_2 & 0 \\ 0 & 0 & dx_2 & 0 & dz_2 & dy_2 \\ a^2-u^2 & -2uw & -2uv & a^2-w^2 & -2vw & a^2-v^2 \end{bmatrix} \times \begin{bmatrix} u_x \\ u_z \\ u_y \\ w_z \\ v_z \\ v_y \end{bmatrix} = \begin{bmatrix} du_1 \\ dv_1 \\ dw_1 \\ dw_2 \\ dv_2 \\ 0 \end{bmatrix} \tag{A6}$$

Notice that the irrotational equations have been used in Eq. (A6). This 6×6 system can easily be solved by a direct elimination method.

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